N90-28015

TIME OPTIMAL MOVEMENT OF COOPERATING ROBOTS

J. M. McCarthy and J. E. Bobrow

Department of Mechanical Engineering
University of California, Irvine
Irvine, CA 92717

Abstract

This paper examines maximizing the speed of movement, along a prescribed path, of the system formed by a set of robot arms and the object they hold. The actuator torques that maximize the acceleration of the system are shown to be determined by the solution to a standard linear programming problem. The combination of this result with the known control strategy for time optimal movement of a single robot arm yields an algorithm for time optimal movement of multiple robot arms holding the same workpiece.

Introduction

The problem of controlling the movement of an actuated closed kinematic chain is a model for coordinating movement of several arms holding the same object, Luh and Zheng 1985, Tarn, et al. 1987, manipulating an object with the fingers of a mechanical hand, Salisbury and Craig 1982, Li et al. 1988, and controlling the posture of a walking machine, Orin and Oh 1981. In each case an important benefit of the closed chain is the distribution of the load at manipulated body, or workpiece, over the actuators of several different robots.

The problem of specifying the joint torques to achieve a specific movement of the chain is underdetermined, which means a variety of joint torque histories perform the same movement. Recent research focusses on using this freedom to balance the load among the actuators by minimizing the total power consumed by the system along the trajectory, Kreutz and Lokshin 1988, Luh and Zheng 1988 and Zheng and Luh 1988.

In this paper the dynamic indeterminacy that appears in the control of two cooperating 3R robots is resolved by seeking the joint torque history that achieves the least transit time along a specified path. This result generalizes the known solution for the open chain case, Bobrow et al. 1985, and joins minimum time path planning research for closed chains with the existing effort for open chains, Gilbert and Johnson 1985, Shiller and Dubowsky 1988, Bobrow 1988, and Rajan 1985.

Dynamics Equations for Cooperating 3R Robots

In this section the equations of motion for the closed chain formed by a pair of cooperating manipulators are derived, focussing on the case of two planar robots. Previous work in this area includes the studies of the cooperation of walking machine legs Orin and Oh 1981, of dual arm robots Luh and Zheng 1985, and Tarn et al. 1987, Kreutz and Lokshin 1988, and of mechanical hands, Nakamura et al. 1986 and Li et al. 1988.

In this derivation we follow Kreutz and Lokshin 1988 and assume that the end effectors of each robot rigidly hold the workpiece. The combination of these end-effectors and the workpiece is viewed as a single moving rigid body. The joints at each end effector provide the forces and moments that move this body. This viewpoint allows us to formulate the dynamics of each arm and the workpiece independently.

Let the two robots be denoted Robot 1 and Robot 2. The joints of Robot 1 are defined by the position vectors $\mathbf{o}_1$, $\mathbf{a}_1$, and $\mathbf{b}_1$, similarly the joints of Robot 2 are $\mathbf{o}_2$, $\mathbf{a}_2$, and $\mathbf{b}_2$, the rotation angles at each of these joints are denoted θ_i, φ_i, and ψ_i, i=1,2, respectively. See Fig. 1. The lengths the first two links of each robot are $K_i=|\mathbf{a}_i-\mathbf{o}_i|$ and $L_i=|\mathbf{b}_i-\mathbf{a}_i|$, i=1,2. The moving body is the rigid link connecting the end joints $\mathbf{b}_1$ and $\mathbf{b}_2$, its length is H. The distance between the base joints $\mathbf{o}_1$ and $\mathbf{o}_2$ is G. For convenience assume that each link is uniform so that its center of mass lies halfway between the joints it contains.

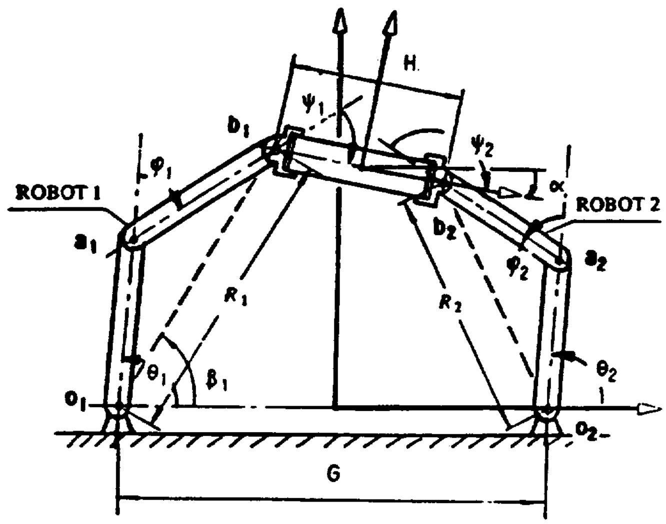

Figure 1. A pair of 3R planar robots holding the same workpiece.

Dynamics of the workpiece. Let $\mathbf{x}=(x,y)^T$ be the position of the center of mass of the workpiece and α be its orientation. If $\mathbf{f}_1=(X_1,Y_1)^T$ and $\mathbf{f}_2=(X_2,Y_2)^T$ are the forces of interaction at the joints $\mathbf{b}_1$ and $\mathbf{b}_2$ and Ψ_1 and Ψ_2 be the motor torques, then the equations of motion of the workpiece are:

$$m\ddot{\mathbf{x}} = \mathbf{f}_1 + \mathbf{f}_2 \ ,$$

$$I\ddot{\alpha} = \mathbf{a}_1 \cdot \mathbf{f}_1 + \mathbf{a}_2 \cdot \mathbf{f}_2 + \Psi_1 + \Psi_2 \qquad (1)$$

where

$$\mathbf{a}_i = \sigma(H/2\sin\alpha, -H/2\cos\alpha)^T, i = 1,2 \ ,$$

and $\sigma=1$ for Robot 1. and $\sigma=-1$ for Robot 2. The vectors $\mathbf{a}_i$ determine the moment of the joint forces about the center of mass. The dot denotes differentiation with respect to time, and m is the mass of the workpiece while I is its moment of inertia about the center of mass.

Eq. (1) can be written in matrix form by adding α to the coordinate vector $\mathbf{x}$, so that $\mathbf{x} = (x,y,\alpha)$ and we obtain

$$[M_0]\ddot{\mathbf{x}} = A_1\mathbf{f}_1 + A_2\mathbf{f}_2 + \tau_0 \ , \tag{2}$$

The torque vector τ_0 is obtained from (1) as

$$\tau_0 = (0,0,\Psi_1 + \Psi_2)^T. \tag{3}$$

and A_1 and A_2 are 3x2 arrays consisting of the 2x2 identity matrix for the first two rows and the vector $\mathbf{a_i}$ in the third row.

The dynamics of each robot. Since the workpiece and end effectors of the robots have been combined into a single body, the equations of motion of each robot reduce to those of a double pendulum with forces applied at the end point $\mathbf{b_i}$. Assemble the joint angles into the vector $\theta_i=(\theta_i, \varphi_i)^T$, and let Θ_i, Φ_i be associated joint torques. The equations of the two robots can be written as

$$[M_i(\theta_i)]\ddot{\theta}_i + \mathbf{h}_i(\theta_i,\dot{\theta}_i) = \tau_i - \mathbf{C}_i, \quad i = 1,2 \tag{4}$$

where

$$\tau_i = (\Theta_i - \Psi_i,\ \ \Phi_i - \Psi_i)^T. \tag{5}$$

$[M_i]$ is the 2x2 mass matrix, $\mathbf{h_i}$ is the vector of Coriolis and gravitational terms, and $\mathbf{C_i}$ arises from the interaction forces at the workpiece. The form of these terms is the same for both robots so we drop the subscript notation:

$$[M_i(\theta)] =$$

$$\begin{bmatrix} I^K + I^L + m^K(K/2)^2 + m^L(L/2)^2 + m^LK^2 + m^LLK\cos\varphi & I^L + m^L(L/2)^2 + \frac{1}{2}m^LLK\cos\varphi \\ I^L + m^L(L/2)^2 + \frac{1}{2}m^LLK\cos\varphi & I^L + m^L(L/2)^2 \end{bmatrix}, \tag{6}$$

$$\mathbf{h}_i(\theta,\dot{\theta}) =$$

$$\begin{pmatrix} -\frac{1}{2}(2\dot{\theta} + \dot{\varphi})\dot{\varphi}m^LLK\sin\varphi - \frac{1}{2}(m^K + 2m^L)gK\cos\theta - m^Lg(L/2)\cos(\theta + \varphi) \\ \frac{1}{2}\dot{\theta}^2 m^LLK\sin\varphi - m^Lg(L/2)\cos(\theta + \varphi) \end{pmatrix}, \tag{7}$$

where the superscripts K, L refer to the cooresponding link with these lentghs.

The vectors $\mathbf{C_i}$ i=1,2 are the generalized joint torques associated with the forces $\mathbf{f_i}=(X_i,Y_i)$ exerted at $\mathbf{b_i}$, and are given by the equations

$$\mathbf{C_i} = [J_i^T]\,\mathbf{f_i}, \quad i=1,2, \tag{8}$$

where

$$[J_i] = \begin{bmatrix} -K_i s\theta_i - L_i s(\theta_i + \varphi_i) & -L_i s(\theta_i + \varphi_i) \\ K_i c\theta_i + L_i c(\theta_i + \varphi_i) & L_i c(\theta_i + \varphi_i) \end{bmatrix}, \qquad (9)$$

is the Jacobian of the i^{th} two link chain. Note s and c denote the sine and cosine functions. The dynamics equations of the workpiece and of the two robots are coupled by the interaction forces at $\mathbf{b_i}$.

The constraint equations. The coordinates, x, y, and α, of the workpiece and the joint angles, θ_i, φ_i, ψ_i, i=1,2, are related by the kinematics equations of the two robots:

$$x = -\sigma G + K_i \cos\theta_i + L_i \cos(\theta_i + \varphi_i) + \sigma(H/2)\cos\alpha,$$
$$y = K_i \sin\theta_i + L_i \sin(\theta_i + \varphi_i) + \sigma(H/2)\sin\alpha, \quad i = 1,2 \qquad (10)$$

where σ=1 for Robot 1 and σ=-1 for Robot 2. Given values for $\mathbf{x}$=(x, y, α), (10) can be solved to determine each of the coordinate vectors $\theta_i=(\theta_i, \varphi_i)$, i=1,2.

We now determine the relation between the joint velocities $\dot{\theta}_i = (\dot{\theta}_i, \dot{\varphi}_i)$ and accelerations $\ddot{\theta}_i$, and the velocity and acceleration, $\dot{\mathbf{x}}$, $\ddot{\mathbf{x}}$, of the workpiece. This conveniently done by introducing the 2 dimensional vector

$$\tilde{\mathbf{x}} = (x + \sigma G - \sigma(H/2)\cos\alpha, Y - \sigma(H/2)\sin\alpha)^T \qquad (11)$$

The derivative of (10) can now be written as

$$\dot{\tilde{\mathbf{x}}} = [J_i]\dot{\theta}_i, \quad i = 1,2 \qquad (12)$$

and

$$\ddot{\tilde{\mathbf{x}}} = [\dot{J}_i]\dot{\theta}_i + [J_i]\ddot{\theta}_i, \quad i = 1,2 \qquad (13)$$

where $[J_i]$ is the Jacobian of each robot, Eq. (9), and $[\dot{J}_i]$ is its derivative with respect to time. For a given position, velocity and acceleration, $\mathbf{x}$, $\dot{\mathbf{x}}$, $\ddot{\mathbf{x}}$ of the workpiece, we can compute $\tilde{\mathbf{x}}$, $\dot{\tilde{\mathbf{x}}}$, $\ddot{\tilde{\mathbf{x}}}$, and solve equations (12) and (13) to determine $\dot{\theta}_i$ and $\ddot{\theta}_i$.

Time Optimal Control

The problem of controlling the cooperating robot pair so that the workpiece traverses a specified path in minimum time is a generalization of work presented in Bobrow et al. 1985. Also see Dubowsky and Shiller 1988 and Shin and McKay 1984.

First, we assemble the equations of motion of the workpiece and the two arms into a single set of equations by introducing the seven dimensional vector $\mathbf{q} = (\mathbf{x}, \theta_1, \theta_2)^T$ --note $\mathbf{x}$ has three components and θ_i each have two components. The equations of motion of the system become

$$[M]\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) = [B]\tau - [C]\mathbf{f}. \tag{14}$$

The 7x7 system mass matrix [M] has $[M_0]$, $[M_1]$ and $[M_2]$ along its diagonal.

The vector $\mathbf{h}(\mathbf{q}, \dot{\mathbf{q}})$ is simply $\mathbf{h} = (0, 0, 0, \mathbf{h}_1, \mathbf{h}_2)^T$ obtained from (7). The system torque vector $\tau = (\Theta_1, \Phi_1, \Psi_1, \Theta_1, \Phi_2, \Psi_2)^T$ is six dimensional and [B] is the 7x6 matrix that maps it to the three torque vectors τ_0, τ_1, τ_2:

$$[B] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \tag{15}$$

The four dimensional vector $\mathbf{f}=(\mathbf{f}_1,\mathbf{f}_2)^T$ represents the interaction forces at the workpiece. The 7x4 matrix [C] is obtained from Eqs. (2) and (8) as

$$[C] = \begin{bmatrix} -A_1 & -A_2 \\ J_1^T & 0 \\ 0 & J_2^T \end{bmatrix} \tag{16}$$

Note that A_1 and A_2 are 3x2 arrays, while J_1^T and J_2^T are 2x2 arrays.

The next step is to introduce a path parameter s which identifies the position of the workpiece as it moves along the specified trajectory. The system equations of motion (14) are written in terms of this parameter. The goal is to determine the maximum acceleration $\ddot{s}$ along the path that is achievable without exceeding the torque limits of the joint motors.

The path parameter. Since it is assumed a path has been specified, the parameterized vector x(s) = (x(s), y(s), α(s))^T is a known function of some parameter s. We seek the function s(t) that minimizes the transit time without exceeding the maximum torque attainable at each joint. Since x(s) is given, we can determine $\tilde{\mathbf{x}}(s)$ from (11), and obtain its derivatives in the form

$$\dot{\bar{x}} = (d\bar{x}/ds)\dot{s}$$
$$\ddot{\bar{x}} = \left(d^2\bar{x}/ds^2\right)\dot{s}^2 + (d\bar{x}/ds)\ddot{s}\ . \tag{17}$$

Using Eqs. (12) and (13) we obtain

$$\dot{\theta}_i = \left[J_i\right]^{-1}(d\bar{x}/ds)\dot{s}$$
$$\ddot{\theta}_i = \left[J_i\right]^{-1}\left\{\left(d^2\bar{x}/ds^2\right)\dot{s}^2 + (d\bar{x}/ds)\ddot{s} - \left[\dot{J}_i\right]\left[J_i\right]^{-1}(d\bar{x}/ds)\dot{s}\right\}. \tag{18}$$

Ultimately, we obtain $\ddot{\mathbf{q}} = \left(\ddot{x},\ \ddot{\theta}_1,\ \ddot{\theta}_2\right)$ in the form

$$\ddot{\mathbf{q}} = \ddot{s}\,\mathbf{q}_1(s,\ \dot{s}) + \mathbf{q}_2(s,\ \dot{s})\ , \tag{19}$$

where $\mathbf{q}_1(s,\ \dot{s})$ is the vector of elements that multiplies $\ddot{s}$ and $\mathbf{q}_2(s,\ \dot{s})$ is the vector of remaining elements.

The equations of motion (14) can now be written in terms of the path parameter, s, as:

$$[M]\,\mathbf{q}_1\ddot{s} + \mathbf{g}(\mathbf{q},\dot{\mathbf{q}}) = [B]\,\tau - [C]\,\mathbf{f}, \tag{20}$$

where

$$\mathbf{g}(\mathbf{q},\dot{\mathbf{q}}) = [M]\mathbf{q}_2 + \mathbf{h}(\mathbf{q},\dot{\mathbf{q}}).$$

For given values of s and $\dot{s}$, Eq. (20) becomes a set of seven linear equations in the eleven unknowns $\ddot{s}$, τ, and $\mathbf{f}$.

The linear programming problem. The optimal control problem now reduces to computing the torques $\tau = (\Theta_1,\ \Phi_1,\ \Psi_1,\ \Theta_2,\ \Phi_2,\ \Psi_2)$ that provide the maximum (or minimum) acceleration $\ddot{s}$ for each position and velocity, $(s,\ \dot{s})$, of the workpiece, subject to the constraint that the equations of motion (20) are satisfied. Note that because the cooperating robots form a three degree of freedom system (7 coordinates - 4 constraints), for a given acceleration $\ddot{s}$ the torques are not in general unique.

The problem of maximizing the acceleration can be posed as a standard problem in Linear Programming, Thie 1979. That is, a vector $\mathbf{y} = (\ddot{s},\ \tau,\ \mathbf{f})^T$ is sought that maximizes (or minimizes) the scalar acceleration

$$\ddot{s} = \mathbf{b}\mathbf{y}, \qquad \mathbf{b} = (1,0,0,0,0,0,0,0,0,0,0) \tag{21}$$

The vector y is subject to the linear constraints due to the equations of motion

$$[A]\,\mathbf{y} = \mathbf{c}, \tag{22}$$

where from (20) we have

$$\begin{aligned}[A] &= \left[[M]\,\mathbf{q}_1, -[B], [C]\right], \\ \mathbf{c} &= -\mathbf{g}.\end{aligned} \tag{23}$$

Bounds on the magnitudes of the components of $\mathbf{y}$ are also part of the standard linear programming problem, which is how torque limits on the motors are introduced. Notice that we can also introduce bounds on the magnitude of the forces acting on the workpiece. The arrays [A], and $\mathbf{c}$ are known constants for every position and velocity $(s, \dot{s})$, so a standard Linear Programming algorithm can be used to determine $\mathbf{y}$ that maximizes $\ddot{s}$, in (21) subject to the constraints in (22).

The solution of this problem provides the set of joint torques τ as well as the interaction forces $\mathbf{f}_1$ and $\mathbf{f}_2$ that yield extreme values for the acceleration, $\ddot{s}$, of the workpiece, for each point $(s, \dot{s})$ in phase space. The result is the ability to compute the acceleration bounds $f(s, \dot{s})$ and $g(s, \dot{s})$,

$$f(s, \dot{s}) \leq \ddot{s} \leq g(s, \dot{s}), \tag{24}$$

such that the dynamics equations (20) are satisfied.

The control strategy. With the ability to compute the maximum and minimum accelerations attainable by the cooperating robot system, the control strategy established in Bobrow et al. 1985 can be applied to achieve a time optimal movement. The essential idea is to always drive the system at its maximum acceleration or deceleration. From this point of view, the problem reduces to the computation of the points at which the shift from acceleration to deceleration and back again occur along the path. To find these switching points, the acceleration equation,

$$\ddot{s} = g(s, \dot{s}), \tag{25}$$

is integrated forward in time from the initial position, and the deceleration equation,

$$\ddot{s} = f(s, \dot{s}), \tag{26}$$

is integrated backward in time from the final position to find when they intersect in phase space. This intersection determines the value of s along the path at which the controller shifts from acceleration to deceleration. In phase space the relation $f(s, \dot{s}) = g(s, \dot{s})$ defines a curve that represents the maximum velocity attainable by the workpiece. It can happen that this maximum velocity constraint is violated before the intersection of the solutions to (25) and (26) occurs. Bobrow et al. 1985 show how to determine intermediate switching points when this occurs, so that time optimal movement is maintained.

Conclusion

In this paper, the time optimal control problem is formulated for the case of two cooperating planar 3R robots. The control strategy is shown to be a generalization of the time optimal control of a single robot arm. The problem of finding minimum time paths for the cooperating 3R arms is a similar generalization of the problem of finding minimum time trajectories for single robot arms. Given the ability to measure transit times for various paths between two desired positions of the workpiece held by several robots, a nonlinear optimization routine can vary the trajectory, while avoiding obstacles in configuration space, until the minimum time path is found.

Bibliography

Bobrow, J. E., 1988, "Optimal Robot Path Planning Using the Minimum-Time Criterion," *IEEE J. Robotics and Automation*, 4(4):443-450.

Bobrow, J. E., Bodduluri, R. M. C., and McCarthy, J. M., 1988, "Path Planning for Cooperating Planar Robots," UCI Mech. Eng. Technical Report, METR-88-12, submitted to the *IEEE J. of Robotics and Automation*.

Bobrow, J. E., Dubowky, S., and Gibson, J. S. 1985. Time-optimal control of robotic manipulators along specified paths. *Int. J. Robotics Research* 4(3):3-17.

Bottema, O., and Roth, B. 1979. *Theoretical kinematics*. Amsterdam: North Holland Publishing Co.

Chen, Y., and Desrochers, A. A. 1988(Philadelphia). Time-optimal control of two-degree of freedom robot arms. *Proc. IEEE Robotics and Automation Conf.* pp. 1210-1215.

Dubowsky, S., and Shiller, Z. 1984 (Udine, Italy). Optimal dynamic trajectories for robotic manipulators. *Proc. V CISM-IFToMM Symp. Theory and Practice of Robots and Manipulators*.

Gilbert, E. G., and Johnson, D. W., 1985, "Distance Funtions and Their Applications to Robot Path Planning in the Presence of Obstacles," *IEEE J. Robotics and Automation*, 1(1):21-30.

Hayati, S. A. 1987(Pasadena). Dynamics and control of coordinated multiple manipulators. *Proc. JPL Workshop on Space Telerobotics*.

Johnson, D. W., and Gilbert, E. G., 1985(Ft. Lauderdale) "Minimum-time robot path planning in the presence of obstacles," *Proc. 24th Conf. on Decision and Control*, pp.1748-1753.

Kahn, M. E., and Roth, B. 1971. The near-minimum-time control of open-loop articulated kinematic chains. *ASME J. Dyn. Sys. Meas. Contr.*, pp. 164-171.

Kerr, J., and Roth, B. 1986. Analysis of multifingured hands. *Int. J. Robotics Research*, 4(4):3-17.

Kreutz, K., and Lokshin, A., 1988(Atlanta). Load balancing and closed chain multiple arm control for a system of several arms grasping a commonly held object. *Proc. American Control Conf.* pp 2148-2155.

Li, Z., Hsu, P., and Sastry, S. 1988(Atlanta). Dynamic coordination of a multiple robotic system with point contact. *Proc. American Control Conf.* pp. 505-510.

Luh, J.Y.S., and Lin, C.S. 1981. Optimal path planning for mechanical manipulators. *ASME J. Dyn. Sys., Meas., Contr.* 12:142-151.

Luh, J.Y.S., and Zheng, Y-F. 1985. Computation of input generalized forces for robots with closed kinematic chain mechanisms. *IEEE J. Robotics and Automation*, 6:60-70.

Luh, J. Y. S., and Zheng, Y.F., 1988(Atlanta). Load distribution between two coordinating robots by nonlinear programming. *Proc. American Control Conf.* pp. 479-482.

McClamrock, N. H., and Huang, H-P. 1985(Seattle). Dynamics of a closed chain manipulator. *Proc. American Control Conference*, pp. 50-54.

Meier, E. B., and Bryson, A. E., 1987(Monterey), "An efficient algorithm for the time-optimal control of a two-link manipulator," *Proc. AIAA Conf. on Guidance and Control*, pp. 204-212.

Nakamura, Y., Nagai, K. and Yoshikawa, T., 1986(Raleigh) Mechanics of coordinative manipulation by multiple robotic mechanisms. *Proc. IEEE Robotics and Automation Conf.* pp 991-998.

Niv, M., and Auslander, D. M. 1984(San Diego). Optimal control of a robot with obstacles. *Proc. Ammerican Contr. Conf.*, 1:280-287.

Orin, D. E., and Oh, S-Y. 1981. Control of force distribution in robotic mechanisms contraining closed kinematic chains. *ASME J. Dyn. Sys., Meas., and Contr.* 102:134-141.

Rajan, V. T., 1985(St. Louis), "Minimum time trajectory planning," *Proc. IEEE Robotics and Automation Conf.*, pp. 759-764.

Salisbury, J.K., and Craig, J. J. 1982. Articulated hands: Force control and kinematics issures. *Int. J. Robotics Research*, 1(1):4-17.

Shiller, Z., and Dubowsky, S. 1988(Philadelphia). Global time optimal motions of robotic manipulator in the presence of obstacles. *Proc. IEEE Robotics and Automation Conf.*, pp. 370-375.

Shin, K. G., and McKay, N. C., 1984(San Diego). Open-loop minimum-time control of mechanical manipulators and its application. *Proc. American Control Conf.*, pp. 296-303.

Tarn, T. J., Bejczy, A. K., and Yun, X. 1987. Design of dynamic control of two cooperating robot arms: closed chain formulation. *Proc. of the IEEE Int. Conf. on Robotics and Automation*, Raleigh, NC. pp. 7-11.

Tarn, T. J., Bejczy, A. K., and Li, Z. F. 1988(Atlanta). Dynamic workspace analysis of two cooperating robot arms. *Proc. American Control Conference*. pp. 489-498.

Thie, P. R. 1979. *An Introduction to Linear Programming and Game Theory* John Wiley, New York, NY. 335pp.

Vanderplaats, G. N. 1984. *Numerical Optimization Techniques for Engineering Design: With Applications*. McGraw-Hill, New York, NY.

Weinreb, A., and Bryson, A. E., 1985, "Optimal control of systems with hard control bounds," *IEEE J. Automatic Control*, AC30(11):1135-1138.

Zheng, Y. F., and Luh, J.Y.S. 1988(Philadelphia). Optimal load distribution for two industrial robots handling a single object. *Proc. IEEE Robotics and Automation Conf.*, pp. 344-349.

COUPLING OF SYMBOLIC AND NUMERIC SYSTEMS